MATRIX AND PROPERTIES

Matrix Type	Definition / Condition	Key Properties
Square Matrix	Same number of rows and columns	Can have determinant, trace, inverse, eigenvalues
Diagonal Matrix	All off-diagonal elements are zero	Diagonal elements may be non-zero
Scalar Matrix	Diagonal matrix with equal diagonal elements	Special case of diagonal matrix
Identity Matrix	Diagonal elements = 1; rest = 0	$A \cdot I = A, I \cdot A = A$
Zero Matrix	All elements are zero	$A + 0 = A, A \cdot 0 = 0$
Symmetric Matrix	$A^T = A$	Diagonalizable, real eigenvalues
Skew-Symmetric	$A^T = -A$	Diagonal elements = 0, eigenvalues are either 0 or purely imaginary
Hermitian Matrix	$A^\dagger = A$ (conjugate transpose = itself)	Complex symmetric analog, eigenvalues are real
Skew-Hermitian	$A^\dagger = -A$	Eigenvalues are purely imaginary or zero
Orthogonal Matrix	$A^T = A^{-1}$, i.e., $AA^T = I$	Preserves vector lengths, determinant = ±1
Unitary Matrix	$A^\dagger = A^{-1}, AA^\dagger = I$	Complex analog of orthogonal matrix, preserves norm
Singular Matrix	Determinant = 0	Not invertible
Non-singular Matrix	Determinant ≠ 0	Invertible
Triangular Matrix	All entries above or below diagonal are zero	Used in LU decomposition; determinant is product of diagonal elements
Normal Matrix	$AA^\dagger = A^\dagger A$	Diagonalizable using unitary matrices

PROBLEMS ON HERMITIAN, SKEW-HERMITIAN, SYMMETRIC, AND SKEW-SYMMETRIC MATRICES

No.	Problem	Solution
1	Determine whether the matrix $A = \begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix}$ is Hermitian.	Take conjugate transpose: $A^\dagger = \begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix}^\dagger = \begin{bmatrix} 2 & -i \\ i & 3 \end{bmatrix}$. This equals A, so **Hermitian**.
2	Is $A = \begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix}$ a Skew-Hermitian matrix?	$A^\dagger = \begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix}^\dagger = \begin{bmatrix} 0 & 2i \\ -2i & 0 \end{bmatrix}^* = \begin{bmatrix} 0 & -2i \\ 2i & 0 \end{bmatrix}$. Thus, $A^\dagger = -A \Rightarrow$ **Skew-Hermitian**.
3	Determine if $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ is symmetric.	$A^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = A \Rightarrow$ **Symmetric**.
4	Find if $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$ is skew-symmetric.	$A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = -A \Rightarrow$ **Skew-symmetric**.
5	Can a real skew-symmetric matrix have non-zero diagonal elements?	No, in skew-symmetric matrices, $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$.
6	Show that the matrix $A = \begin{bmatrix} 0 & 2+i \\ -2-i & 0 \end{bmatrix}$ is skew-Hermitian.	$A^\dagger = \begin{bmatrix} 0 & -2+i \\ 2-i & 0 \end{bmatrix} = -A \Rightarrow$ **Skew-Hermitian**.
7	Show that every square matrix can be written as the sum of a Hermitian and a skew-Hermitian matrix.	For any matrix A, write: $A = \frac{1}{2}(A + A^\dagger) + \frac{1}{2}(A - A^\dagger)$. First part is Hermitian, second part is skew-Hermitian.
8	Check if the matrix $A = \begin{bmatrix} 1+i & 2 \\ 2 & 1-i \end{bmatrix}$ is Hermitian.	$A^\dagger = \begin{bmatrix} 1-i & 2 \\ 2 & 1+i \end{bmatrix} = A \Rightarrow$ **Hermitian**.
9	Find the Hermitian and skew-Hermitian parts of $A = \begin{bmatrix} 2 & 3+i \\ 3-i & 4 \end{bmatrix}$	Hermitian part = $\frac{1}{2}(A + A^\dagger) = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ Skew-Hermitian part = $\frac{1}{2}(A - A^\dagger) = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$
10	Determine if the matrix $A = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix}$ is Hermitian.	$A^\dagger = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix}^* = \begin{bmatrix} 1 & -2i \\ 2i & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix} = A$, hence **Hermitian**.

EIGENVALUES AND EIGENVECTORS

S.No.	Property	Explanation	Remarks/Examples
1	**Definition**	If $A\vec{x} = \lambda\vec{x}$, then λ is an eigenvalue and $\vec{x} \neq 0$ is an eigenvector.	$A \in \mathbb{R}^{n\times n}, \vec{x} \in \mathbb{R}^n$
2	**Characteristic Equation**	Eigenvalues are solutions of $\det(A - \lambda I) = 0$.	A polynomial equation of degree n.
3	**Number of Eigenvalues**	An $n \times n$ matrix has n eigenvalues (including complex and repeated).	Can be real or complex.
4	**Linearly Independent Eigenvectors**	Eigenvectors corresponding to **distinct** eigenvalues are linearly independent.	Important for diagonalization.
5	**Eigenvalues of Diagonal Matrix**	Are the diagonal entries themselves.	E.g. $D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow \lambda = 2,3$
6	**Eigenvalues of Triangular Matrix**	Also lie on the diagonal.	Works for both upper and lower triangular matrices.
7	**Trace**	Sum of eigenvalues = Trace of matrix.	$\text{tr}(A) = \sum \lambda_i$
8	**Determinant**	Product of eigenvalues = Determinant of the matrix.	$\det(A) = \prod \lambda_i$
9	**Eigenvalues of A^{-1}**	If λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.	Provided A is invertible.
10	**Eigenvalues of A^T**	Same as those of A.	A and A^T have identical characteristic polynomials.
11	**Eigenvalues of scalar multiple kA**	Are $k\lambda_i$, if λ_i are eigenvalues of A.	Example: $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, 2A \Rightarrow \lambda = 2,4$
12	**Eigenvectors remain same under scalar multiplication**	If $A\vec{x} = \lambda\vec{x}$, then $(kA)\vec{x} = k\lambda\vec{x}$.	Eigenvector is unchanged.
13	**Eigenvalues of A^n**	Are λ^n, if λ is an eigenvalue of A.	For powers of matrices.
14	**Orthogonality (Symmetric Matrices)**	Eigenvectors of real symmetric matrices are orthogonal.	Important in diagonalization.
15	**Complex Eigenvalues**	If A has real entries, then complex eigenvalues occur in conjugate pairs.	E.g. $\lambda = a + bi \Rightarrow \bar{\lambda} = a - bi$ also an eigenvalue.

BESSEL, LEGENDRE, HERMITE, AND LAGUERRE POLYNOMIALS

Property	**Bessel** $J_n(x)$	**Legendre** $P_n(x)$	**Hermite** $H_n(x)$	**Laguerre** $L_n(x)$
Type	Special function (non-polynomial)	Orthogonal polynomial	Orthogonal polynomial	Orthogonal polynomial
Order	Integer or real n	Integer n	Integer n	Integer n
Domain	$x \in \mathbb{R}, \mathbb{C}$	$x \in [-1,1]$	$x \in \mathbb{R}$	$x \in [0, \infty)$
Weight Function	$w(x) = x$	$w(x) = 1$	$w(x) = e^{-x^2}$	$w(x) = e^{-x}$
Orthogonality Relation	$\int_0^1 x J_n(\alpha_{n,m}x) J_n(\alpha_{n,k}x) dx = 0$ for $m \neq k$	$\int_{-1}^1 P_n(x) P_m(x)\, dx = 0$ for $n \neq m$	$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\, dx = 0$	$\int_0^{\infty} e^{-x} L_n(x) L_m(x)\, dx = 0$
Differential Equation	$x^2 y'' + xy' + (x^2 - n^2)y = 0$	$(1-x^2)y'' - 2xy' + n(n+1)y = 0$	$y'' - 2xy' + 2ny = 0$	$xy'' + (1-x)y' + ny = 0$
Generating Function	$e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} J_n(x) t^n$	$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x) t^n$	$e^{2xt-t^2} = \sum_{n=0}^{\infty} \frac{H_n(x)}{n!} t^n$	$\frac{e^{-xt/(1-t)}}{1-t} = \sum_{n=0}^{\infty} L_n(x) t^n$
Recurrence Relation	$J_{n-1}(x) + J_{n+1}(x) = \frac{2n}{x} J_n(x)$	$(n+1)P_{n+1} = (2n+1)xP_n - nP_{n-1}$	$H_{n+1} = 2xH_n - 2nH_{n-1}$	$L_{n+1} = \frac{(2n+1-x)L_n - nL_{n-1}}{n+1}$
Rodrigues' Formula	–	$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2-1)^n]$	$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}[e^{-x^2}]$	$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}[e^{-x} x^n]$
Applications	Cylindrical systems, EM waves	Spherical systems, potential theory	Quantum oscillator, probability	Hydrogen atom, quantum systems
Zeros	Infinite, real and positive	Real in $(-1,1)$	Real, symmetric about 0	Real, in $(0, \infty)$
Normalization	$J_0(0) = 1$	$P_n(1) = 1$	Leading term = 2^n	$L_n(0) = 1$

Property	Bessel $J_n(x)$	Legendre $P_n(x)$	Hermite $H_n(x)$	Laguerre $L_n(x)$
Terms	$J_0(x) = 1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \cdots$ $J_1(x) = \frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \cdots$ $J_2(x) = \frac{x^2}{8} - \frac{x^4}{96} + \frac{x^6}{3072} - \cdots$ $J_3(x) = \frac{x^3}{48} - \frac{x^5}{768} + \cdots$ $J_4(x) = \frac{x^4}{384} - \frac{x^6}{9216} + \cdots$ $J_5(x) = \frac{x^5}{3840} - \cdots$	$P_0(x) = 1$ $P_1(x) = x$ $P_2(x) = \frac{1}{2}(3x^2 - 1)$ $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ $P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$	$H_0(x) = 1$ $H_1(x) = 2x$ $H_2(x) = 4x^2 - 2$ $H_3(x) = 8x^3 - 12x$ $H_4(x) = 16x^4 - 48x^2 + 12$ $H_5(x) = 32x^5 - 160x^3 + 120x$	$L_0(x) = 1$ $L_1(x) = -x + 1$ $L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$ $L_3(x) = \frac{1}{6}(-x^3 + 9x^2 - 18x + 6)$ $L_4(x) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$ $L_5(x) = \frac{1}{120}(-x^5 + 25x^4 - 200x^3 + 600x^2 - 600x + 120)$

Binomial, Poisson, and Normal Distributions

This guide compares the **Binomial**, **Poisson**, and **Normal** distributions in terms of their introduction, formulas, statistical moments, modes, characteristic functions, cumulants, cumulative functions, and applications.

Introduction

- **Binomial Distribution**: Describes the number of successes in a fixed number of independent Bernoulli trials (yes/no experiments) with the same probability of success.
- **Poisson Distribution**: Models the number of times an event occurs in a fixed interval of time or space, assuming events happen independently and at a constant mean rate.
- **Normal Distribution**: Represents a continuous probability distribution, known for its bell-shaped curve; it is used for distributions of sums (or averages) of many small, independent effects.

Distribution Formulas

Property	Binomial	Poisson	Normal
PDF/PMF	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$	$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
Parameters	n: trials, p: success prob.	λ: mean count	μ: mean, σ^2: variance

Power Series Expansion

- **Binomial**: Based on binomial theorem, directly tied to discrete trials.
- **Poisson**: Derived as a limiting case of the binomial when $n \to \infty$, $p \to 0$, with $np = \lambda$ fixed.
- **Normal**: Taylor expansion in its density exponents; arises as the limiting case of binomial or Poisson by the Central Limit Theorem.

First Four Moments

Order	Binomial	Poisson	Normal
Mean	np	λ	μ
Variance	$np(1-p)$	λ	σ^2
Skewness	$\frac{1-2p}{\sqrt{np(1-p)}}$	$\frac{1}{\sqrt{\lambda}}$	0
Kurtosis (excess)	$\frac{1-6p(1-p)}{np(1-p)}$	$\frac{1}{\lambda}$	0

Mode(s)

- **Binomial**: $\lfloor (n+1)p \rfloor$ (Mode is not always unique for integer values.)
- **Poisson**: $\lfloor \lambda \rfloor$
- **Normal**: The mode is at μ (also the mean and median).

Moment Generating Function (MGF)

Property	Formula
Binomial	$M(t) = (1-p+pe^t)^n$
Poisson	$M(t) = \exp(\lambda(e^t-1))$
Normal	$M(t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2)$

Cumulants

- **Binomial**: rth cumulant is $n\frac{d^r}{dt^r}\ln(1-p+pe^t)|_{t=0}$
- **Poisson**: All cumulants are equal to λ.
- **Normal**: 1st is μ, 2nd is σ^2, higher orders are zero.

Cumulative Distribution Function (CDF)

- **Binomial**: $F(k) = \sum_{i=0}^{k}\binom{n}{i}p^i(1-p)^{n-i}$
- **Poisson**: $F(k) = \sum_{i=0}^{k}\frac{\lambda^i e^{-\lambda}}{i!}$
- **Normal**: $F(x) = \frac{1}{2}\left[1+\text{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$ (No closed-form except using the error function.)

Typical Applications

Distribution	Typical Application Examples
Binomial	Quality control (defective products), coin flips, surveys
Poisson	Arrival of calls at a call center, radioactive decay, rare event modeling
Normal	Heights, test scores, measurement errors in science

Summary Table

Feature	Binomial	Poisson	Normal
Type	Discrete	Discrete	Continuous
Range	$0,1,\ldots,n$	$0,1,\ldots,\infty$	$(-\infty,\infty)$
Skewness	Varies with p	Decreases as λ increases	0 (symmetric)
Limiting Relation	-	Limit of Binomial	Limit of Binomial/Poisson as $n\to\infty$

MAXWELL'S EQUATIONS

Maxwell's Equations (in differential form)

Maxwell's equations unify electricity, magnetism, and optics into one electromagnetic theory. There are **four equations**, named after **James Clerk Maxwell**, based on experimental laws:

S.No.	Equation	Name	Empirical Basis
1.	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	**Gauss's Law for Electricity**	Coulomb's Law
2.	$\nabla \cdot \vec{B} = 0$	**Gauss's Law for Magnetism**	No magnetic monopoles
3.	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	**Faraday's Law of Induction**	Faraday's experiments
4.	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	**Ampère–Maxwell Law**	Ampère's Law + Maxwell's Displacement Current

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1. Gauss's Law for Electricity

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

- **Meaning**: Electric field lines diverge from electric charges.
- ρ: charge density (C/m^3)
- $\vec{E}$: Electric field (V/m)
- ε_0: Permittivity of free space (8.854×10^{-12} F/m)

Units: LHS: $\nabla \cdot \vec{E} \Rightarrow \mathrm{V/m^2}$ RHS: $\rho/\varepsilon_0 \Rightarrow \mathrm{C/(m^3)/F/m = V/m^2}$

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2. Gauss's Law for Magnetism

$$\nabla \cdot \vec{B} = 0$$

- **Meaning**: No isolated magnetic monopoles; magnetic field lines are always **closed loops**.
- $\vec{B}$: Magnetic flux density (Tesla)

Units: LHS: $\nabla \cdot \vec{B} \Rightarrow \mathrm{T/m = kg/(s^2 \cdot A)}$ RHS: 0 (dimensionless) — implies conservation.

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3. Faraday's Law of Induction

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- **Meaning**: Time-varying magnetic fields induce **electric fields** (EMF).
- Basis: Faraday's experimental observations

Units: LHS: $\nabla \times \vec{E} \Rightarrow \mathrm{V/m^2}$ RHS: $-\partial \vec{B}/\partial t \Rightarrow \mathrm{T/s = V/m^2}$

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4. Ampère–Maxwell Law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- **Meaning**: Magnetic fields are produced by **currents and changing electric fields**
- First term: Ampère's Law
- Second term: **Displacement current** added by Maxwell

Units:

- LHS: $\nabla \times \vec{B} \Rightarrow \mathrm{T/m = kg/(s^2 \cdot A)}$
- RHS:
  - $\mu_0 \vec{J}$: $\mathrm{(H/m) \cdot A/m^2 = T/m}$
  - $\mu_0 \varepsilon_0 \, \partial \vec{E}/\partial t = \mathrm{T/m}$

Summary Table with Units

Equation	Vector Form	Physical Meaning	Units
1. Gauss's Law (E)	$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$	Electric flux divergence equals charge density	V/m^2
2. Gauss's Law (B)	$\nabla \cdot \vec{B} = 0$	No magnetic monopoles	T/m
3. Faraday's Law	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Time-varying B induces E	V/m^2
4. Ampère–Maxwell Law	$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	Currents and time-varying E induce B	T/m

Mechanics of Particles

Mechanics of particles deals with the motion and interaction of particles (objects with mass but negligible size). It includes the study of **force, momentum, energy**, and their **conservation laws**.

Conservation of Linear Momentum

Principle:

In the absence of external force, the **total linear momentum** of a system remains **constant**.

$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}} \quad \text{if } \vec{F}_{\text{ext}} = 0$

Example:

A **0.05 kg bullet** is fired from a **1 kg gun** with velocity **v = 100 m/s**. Find the recoil velocity of the gun.

Solution:

Initial momentum = 0

Final momentum: $0 = m_{\text{gun}} \cdot v_{\text{gun}} + m_{\text{bullet}} \cdot v_{\text{bullet}}$

$$0 = 1 \cdot v_{\text{gun}} + 0.05 \cdot 100 \Rightarrow v_{\text{gun}} = -5 \text{ m/s}$$

Gun recoils at 5 m/s in the opposite direction.

Conservation of Angular Momentum

Principle:

If no external torque acts on a system, its **angular momentum** remains constant. $L = \vec{r} \times \vec{p}, \quad \vec{\tau}_{\text{ext}} = 0 \Rightarrow \frac{dL}{dt} = 0$

Example:

A skater has **moment of inertia** $I = 4$ kg·m^2 and **angular velocity** $\omega = 2$ rad/s. When arms are pulled in, $I = 2$ kg·m^2. What is the new angular velocity?

Solution:

$L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2 \Rightarrow 4 \cdot 2 = 2 \cdot \omega_2 \Rightarrow \omega_2 = 4$ rad/s

New angular velocity = 4 rad/s.

Conservation of Energy

Principle:

In a conservative system (no friction or loss), **mechanical energy** (kinetic + potential) is conserved. $E = KE + PE =$ constant

Example:

A **2 kg** object falls from **10 m**. What is its speed just before hitting the ground?

Solution:

Initial PE = $mgh = 2 \cdot 9.8 \cdot 10 = 196\,J$ Final KE = $\frac{1}{2}mv^2 = 196 \Rightarrow v = 14$ m/s

Speed = 14 m/s.

QUICK REVISION POINTS

Mechanics of Particles and Systems of Particles

- **Particle**: Idealized object with mass but no size.
- **System of particles**: A collection of interacting particles.
- **Newton's laws** apply individually to each particle.
- **Centre of mass**: Average position of the mass in a system.
- **Linear momentum (P)** of system = total mass × velocity of center of mass.
- **Angular momentum (L)** of system = conserved if net external torque is zero.
- **Total energy** = Kinetic + Potential (conserved in isolated systems).

Constraints and Generalized Coordinates

- **Constraint**: Restriction on motion (e.g., a bead on a wire).
- **Generalized coordinates ($q_1, q_2, ..., q_n$)**: Minimal set of independent coordinates to describe a system.
- **Degrees of freedom**: Number of independent coordinates.
- **Holonomic system**: Constraints can be expressed as equations (e.g., $f(q,t) = 0$).
- **Non-holonomic**: Inequalities or differential constraints (e.g., rolling without slipping).
- **Scleronomic**: Constraints independent of time.
- **Rheonomic**: Constraints explicitly time-dependent.

Conservation Laws

- **Conservation of Energy**: Total mechanical energy is conserved if the force is conservative.
- **Conservation of Linear Momentum**: Conserved if **no external force** acts on the system.
- **Conservation of Angular Momentum**: Conserved if **no external torque** acts.
- **Conservative forces**: Derivable from potential energy $\vec{F} = -\nabla V$.
- **Non-conservative forces**: Path-dependent (e.g., friction).

Lagrangian Formalism

- **Lagrangian**: $L = T - V$, where T is kinetic and V is potential energy.
- **Lagrange's Equations**:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

- **D'Alembert's Principle**: Introduces virtual work to derive equations of motion.

Applications:

- **Simple Pendulum**: One degree of freedom (angle θ).
- **Atwood's Machine**: Two-mass pulley system.
- **Harmonic Oscillator**: Restoring force proportional to displacement.
- **Electrical LC Circuit**: Analogy: Charge ↔ Position, Current ↔ Velocity.

Hamiltonian Formalism

- **Hamiltonian**: $H = \sum_i p_i \dot{q}_i - L$, often equals **total energy**.
- **Hamilton's Equations**:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

- **Cyclic (ignorable) coordinates**: If $\partial L / \partial q_i = 0$, then p_i is conserved.

- **Variational Principle (Hamilton's Principle)**: Motion follows a path that minimizes the **action**:

$$\delta \int_{t_1}^{t_2} L\, dt = 0$$

- **Principle of Least Action**: Foundation of all classical mechanics.
- **Canonical Transformations**: Preserve Hamiltonian structure of equations.
- **Liouville's Theorem**: Phase space volume is conserved in time.

🌀 Rigid Body Dynamics

- **Rigid Body**: Distance between particles remains constant.
- **Moment of Inertia Tensor**: Describes how mass is distributed with respect to rotational axes.
- **Euler's Angles (θ, ϕ, ψ)**: Specify orientation of body in space.

📉 Euler's Equations of Motion

- For a rigid body rotating about a point:

$$I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 = N_1$$
$$I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 = N_2$$
$$I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 = N_3$$

- Derived in body-fixed frame (rotating axes).

□ Symmetrical Top under Gravity

- Symmetric: $I_1 = I_2 \neq I_3$
- Fixed point at one end, center of mass offset.
- Uses **Euler angles**.
- Motions:
  - **Spin**: About symmetry axis.
  - **Precession**: Around vertical axis.
  - **Nutation**: Wobbling of axis.
- Lagrangian:

$$L = \frac{1}{2}I_1(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + \frac{1}{2}I_3(\dot{\psi} + \dot{\phi}\cos\theta)^2 - mgl\cos\theta$$